V-STARS/M System Accuracy Test Results

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Abstract

In April, 2000, Boeing conducted a comprehensive acceptance test of Geodetic Services Inc.'s (GSI's) V-STARS/M real-time photogrammetric measurement system. The purpose of the test was to establish the three-dimensional coordinate measurement accuracy of the V-STARS/M system under various operating modes and conditions. Two tests were conducted; the "apex angle test" determined how the system accuracy depends on the intersection angle between the two cameras. The "probe test" determined the system accuracy using the hand-held probes provided with the system. The results were compared to an independent measurement of the test object established by a laser tracker. Every practical step was taken to ensure the laser tracker measurement was of the highest possible accuracy. This report describes the tests, measurement results, accuracy achieved, and a set of recommended best practices.

System Description

GSI's V-STARS/M system is a real-time, photogrammetric coordinate measurement system. It primarily consists of a notebook computer, and two or more ultra high-resolution digital cameras. The system measures the 3-dimensional coordinates of points of interest by intersecting the lines of sight from the cameras to the points using a process called triangulation. The complete system is shown in Figure 1.

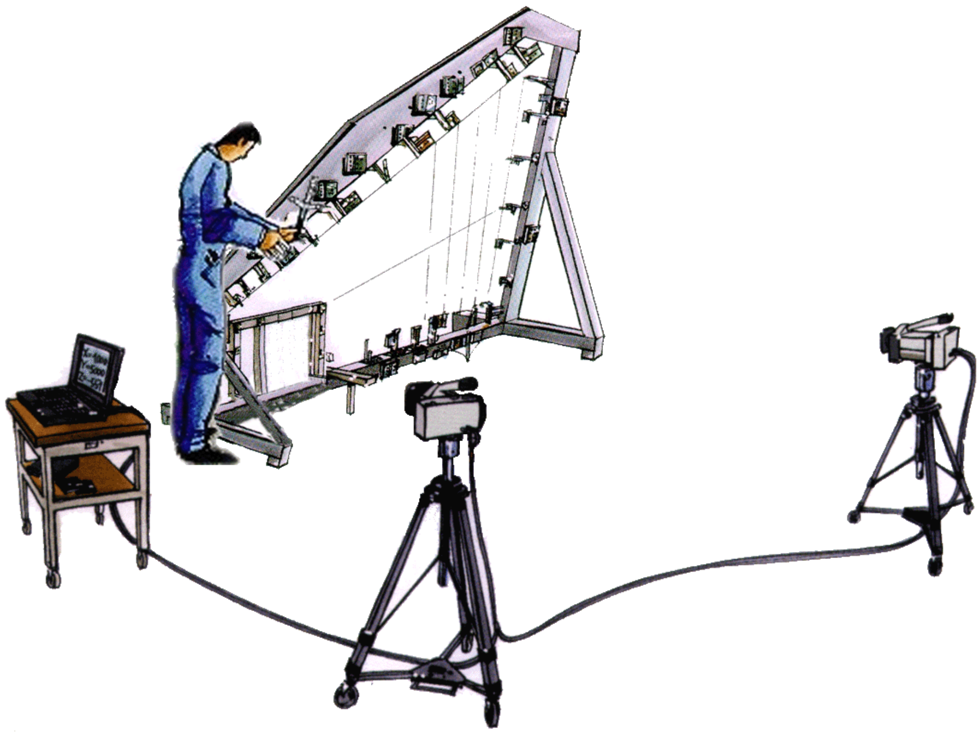

Figure 1. V-STARS/M System

The V-STARS/M system can measure points in several ways. It can measure retro-reflective targets that are applied to the object. These targets are small, lightweight, inexpensive and easy to apply. In some cases, the system can also measure high-contrast light dots that are projected onto the object using GSI's PRO-SPOT target projector. Finally, the system can also measure points without targeting using a small, wireless hand-held probing tool that has a touch point. Examples of several probes are shown in Figure 2. Retro-reflective targets on the probe are measured to produce the coordinates of the touch point. Multiple probes (up to 16) can be used on a measurement and the software automatically detects which probe is being used. Typically a measurement cycle takes one to two seconds.

Figure 2. V-STARS/M System Probes

The V-STARS/M system includes GSI's V-STARS/S single-camera system. The S system uses one of the M system cameras to take multiple pictures of an object from different locations. These pictures are then automatically processed to yield the coordinates of points on an object.

The V-STARS/S system is not real-time so it can only measure static objects and targeted points. However, it is well suited to high accuracy measurement of large, complicated objects since practically any number of pictures can be taken and processed. In addition, the photography is usually quick, so production downtime is low and temperature effects are minimized. Boeing conducted a separate accuracy test of the V-STARS/S system. (see references 1,2). The use of the V-STARS/S camera in a typical measurement is shown in Figure 3.

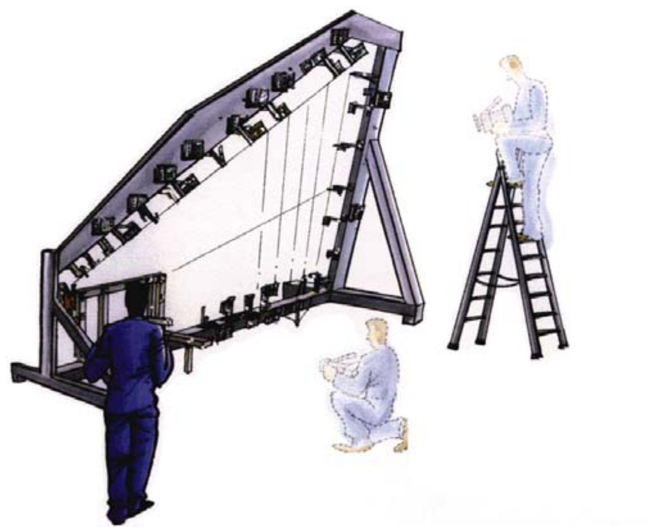

Figure 3. V-STARS/S System in Typical Operation

One of the most powerful features of the V-STARS/M system is its ability to measure in unstable environments. This is accomplished by placing some targets on the object that will ultimately serve as reference points for the M system in the unstable mode. The coordinates for these points are typically established by a one-time V-STARS/S measurement. Then, the V-STARS/M system uses these points to calculate the position and orientation of the cameras each time a measurement is made. Thus, movement of the cameras (and/or the object) is accounted for on every picture and has no effect on system accuracy. This capability combined with the synchronized camera strobes allows the system to make accurate measurements even in areas where vibrations are present.

The V-STARS/M testing described in this report used the unstable mode for all the tests. The V-STARS/M system can also operate in a stable mode where no targeting of the object is required (and consequently the cameras and object must be stable throughout the measurement). However, this mode was not tested since it is not used for most Boeing applications.

Description of Test Object

The testing was performed on a large, stable granite surface plate that was 12 feet long and 4 feet wide. In order to get as large a 3-dimensional test volume as practicable, the support table for the granite plate was also used and several large angle-irons were attached to the top surface of the granite. The resulting test volume was 11.7 by 4.4 by 3.9 feet (141 x 53 x 47 inches).

The coordinate system for the testing was defined as shown in Figure 4. The origin is at the top, left corner of the granite plate (as viewed from the cameras); the X axis is along the long side of the table; the Y axis is along the short side of the table; the Z axis is up, out of the table.

Figure 4. Granite plate used for testing

Description of Accuracy Standard (using laser tracker)

To establish the measurement accuracy of a system, one can compare the measurement results to an established standard. The standard should be of considerably higher accuracy than the measurement system under test. In addition, the standard and the system under test should measure the same physical features (this is called "target duplication"). Finally, it is important that variations caused by temperature should be minimal.

Boeing went to great efforts to establish a very accurate standard for the tests using a laser tracker. For example, the laser tracker used a certified environmental monitor to compensate for atmospheric effects.

Although the laser tracker could have measured the object in a single setup, three setups were used to improve the accuracy. The three surveys were then rigorously combined using a bundle adjustment to get the final values for the comparisons. The accuracy of the laser tracker survey was then regarded as perfect in the M system accuracy analysis. This simplified the analysis by allowing the differences between the two systems to define the accuracy of the M system. To the extent the laser tracker survey was not perfect, it simply means that the M system accuracies are actually better than shown in this report, and therefore the estimates are conservative.

Two types of target holders were used for the test points. Bushings were used to hold individual retro-reflective tooling targets. “Nest” target holders designed to hold a 1.5 inch diameter spherical corner cube were used for points measured with the probe. Examples of each are shown in Figure 5. The test points were well distibuted throughout the volume of the test object.

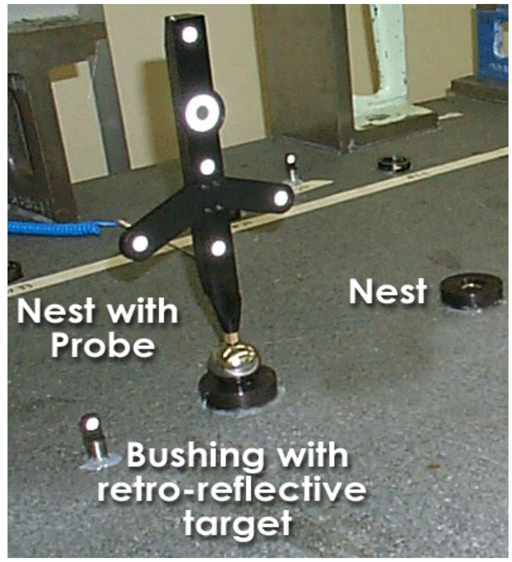

Figure 5. Bushings and nests

The laser tracker measured the test points using two different corner cubes. A ½ inch diameter spherically mounted corner cube was used to measure all the bushings. The corner cube had an adapter that offset its center ½ inch from the base. A 1.5 inch diameter spherical corner cube was used for all the nest points. For the M testing, standard ½ inch offset retro-reflective targets were used to measure the bushed holes, and a 1.5 inch diameter sphere tip was used to measure the nest points. This ensured there was minimal target offset errors between the two measurement systems, and allowed their coordinates to be compared directly. Targeting variations are estimated to be the largest uncompensated (systematic and random) error encountered in the testing.

Test Descriptions

Two tests of the M system were conducted. Both used the unstable mode of operation. In this mode, known points on the object are used to solve for each camera’s position and orientation on each measurement. Thus, movement of the camera (or object) was compensated for on the measurement.

The known points were established using the V-STARS/S mode of operation. Pictures were taken of the object from several different locations using one of the cameras. These were then processed to produce the coordinates of the targeted points on the object. Twenty-five well-distributed points were then used as unstable-mode reference points for all the subsequent M tests.

Apex angle Test

Since the V-STARS/M system is based on triangulation, its accuracy is dependent on the intersection angle of the two cameras. This angle is called the “apex angle”. The apex angle test determined how the accuracy of the M system varied as the apex angle between the two cameras was changed from 15 to 105º in 15º increments. The setup of the cameras for each apex angle is shown in Figure 6.

For each test, the cameras were separated by the apex angle (as marked on the table, see Figure 7), and placed far enough back so the object comfortably fit within the field of view of either camera. The camera's built-in TTL (Thru' The Lens) viewfinder was helpful for doing this. Typically, the cameras were about twelve feet away from the center of the object.

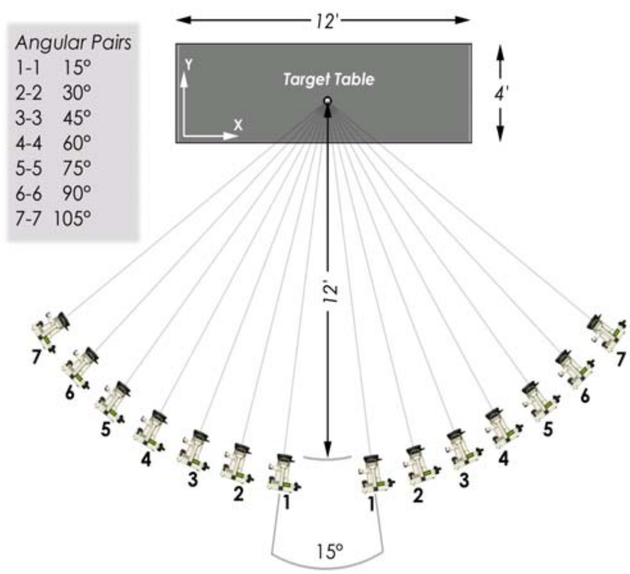

Figure 6. Camera Locations for Apex-angle Test

Figure 7. Apex-angle Test Setup

Seventy-five retro-reflective targets were used for the apex-angle testing. As mentioned earlier, twenty-five of the targets were used to calculate the position and orientation of the cameras on each measurement so camera movement was of no consequence. The remaining fifty targets were triangulated for the apex-angle test. These targets were well distributed over the entire measurement area. At each apex angle, the targets were measured ten times (yielding 500 total measurements). To verify the claim that camera and/or object movement cause no degradation of accuracy, the cameras were deliberately moved slightly for the last five measurements at each apex angle. The test produced 3500 measurements overall (500 at each of the seven different apex angles).

Although the testing could have used a probe, using individual targets was much faster since all fifty of the apex-angle targets were measured at the same time, and measurement time for all 75 points (25 reference and 50 measured) took approximately two seconds. The set of ten measurements at each apex angle was completed in a few minutes (including the time to move the cameras for the last five measurements), and the entire test took under an hour.

This test then served three purposes. It assessed the accuracy variation with apex angle, assessed the system's performance when measuring individual target points, and verified that the system performance is not degraded in the unstable operating mode (i.e. by camera and/or object movement). A separate probe test was done as described later to assess the accuracy of measurements with the probe.

Apex Angle Test Results

Each of the 50 target points used in the apex-angle testing was measured 70 times (10 measurements per apex angle and 7 different apex angles) for a total of 3500 measurements. For the analysis, the differences between the 50 laser-tracker values for the target points and all their measured values were computed for each coordinate (X,Y,Z). No transformation was required before calculating the differences since the measured points were already in the laser-tracker coordinate system by virtue of the earlier V-STARS/S survey of the 25 unstable-mode reference points.

In addition, the RSS (square root of the sum of the squares) of the three coordinate differences for each measurement were calculated. The formula for the RSS of a point is:

$$RSS = (X_{difference}^{2} + Y_{difference}^{2} + Z_{difference}^{2})^{1/2}.$$

The RSS then is the length of the difference vector between the laser-tracker value and the measured value. The RSS is a useful statistic that combines together the separate coordinate differences into one meaningful quantity.

The RSS differences are summarized and presented graphically in Figure 8 below. Three statistics were calculated and plotted for each of the seven apex angles. These are the average of the RSS differences, the standard deviation of the RSS differences, and a derived statistic that is the average plus two times the standard deviation. As is explained later, this statistic can be used as a reasonably good approximation for the 95% confidence level of the measurement.

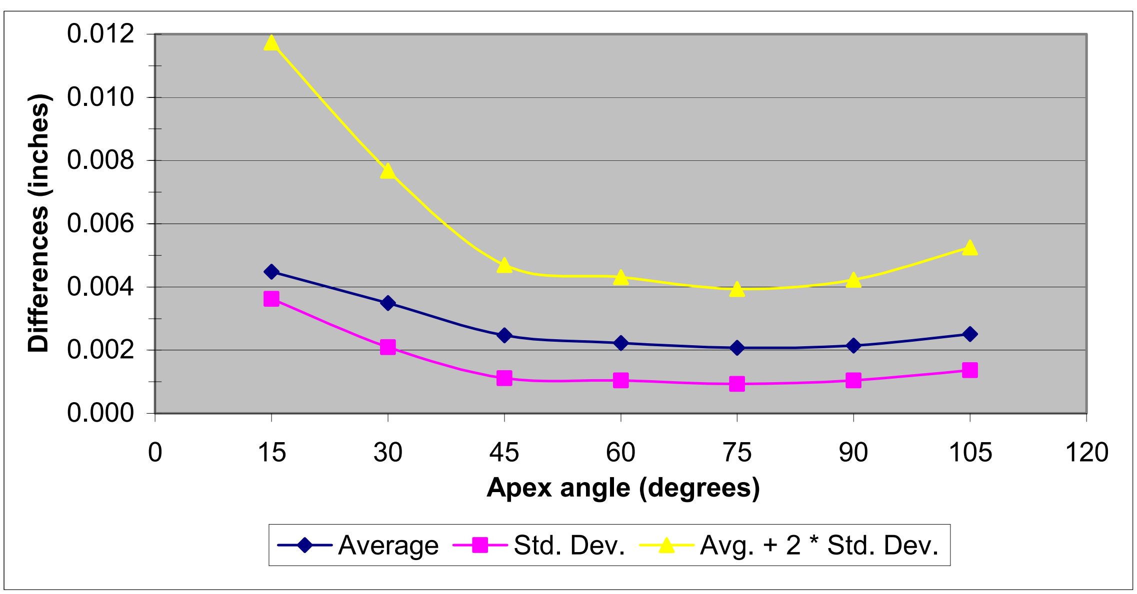

Figure 8. RSS Statistics for Apex-angle Test

The results for the RSS calculation show the influence of apex angle on overall system accuracy. Each plotted point on the graph shows the statistics for the 500 measurements at each of the seven different apex angles. Note these statistics increase significantly at the 30 and 15º apex angles, and also increase slightly at the 105º angle. At the other apex angles the statistics are similar.

Looking at the results in terms of the individual coordinate differences further illustrates the relationship between apex angle and accuracy. The standard deviations of the differences in each coordinate are shown graphically in Figure 9.

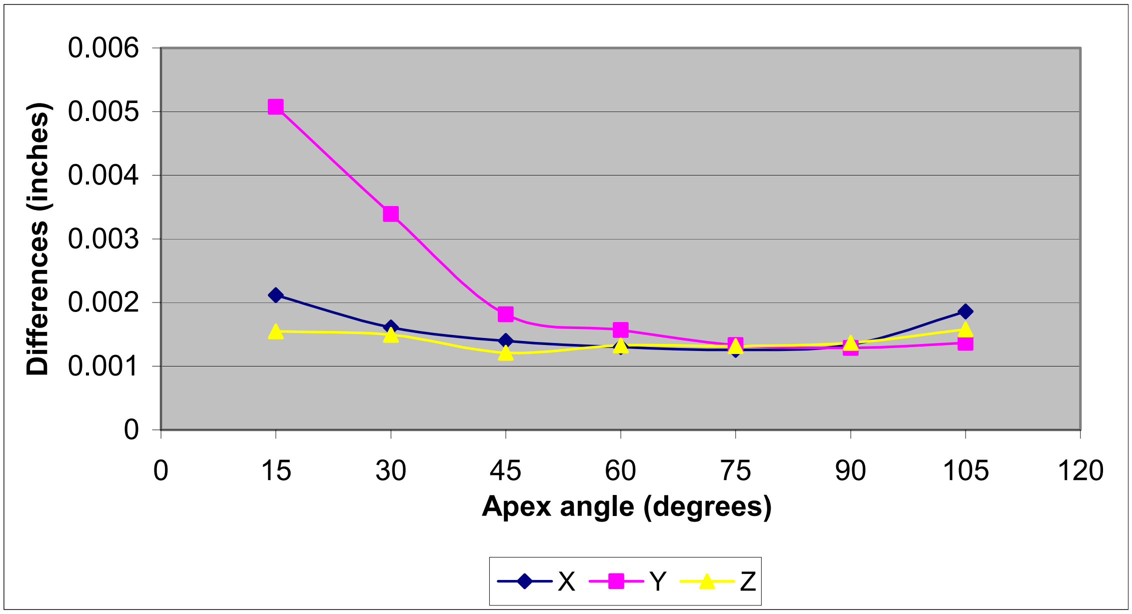

Figure 9. Apex Angle Test Standard Deviations for X, Y and Z Coordinates

By separating the differences into their component values, the apex-angle accuracy dependence is seen primarily as a Y-coordinate effect. This is due to the definition of the coordinate system. The Y-axis is perpendicular to the camera baseline. As the apex angle widens past 90º, the dependence in the Y direction is low and the dependence in the X direction (parallel to the camera baseline) increases although nowhere near as dramatically as the Y increase. This behavior is expected since the V-STARS system is a triangulation system. The effect is illustrated in Figure 10 below.

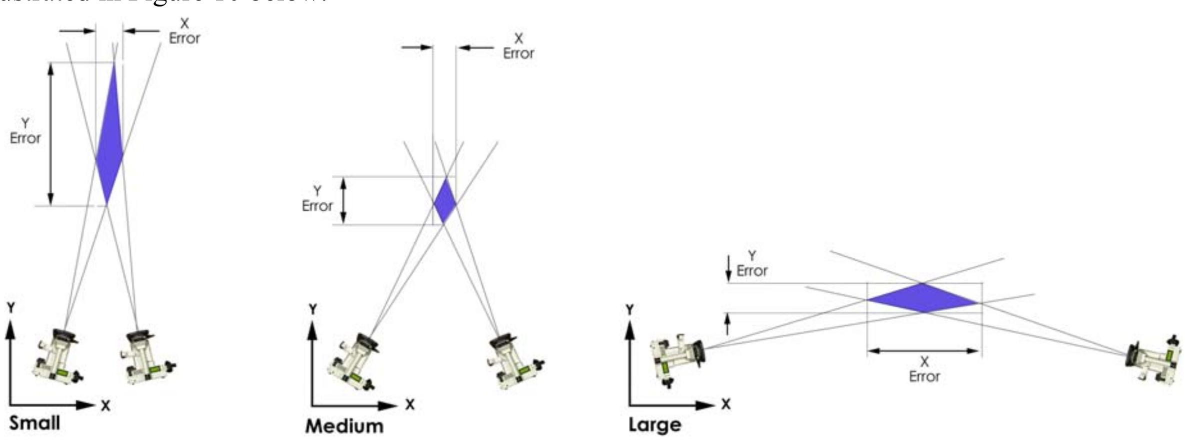

Figure 10. Apex-angle Error Envelope Diagram

The error envelope is shown for small, medium and large apex angles. The lines emanating from the cameras represent the error bounds on the image measurement of the point. The intersection area of the two sets of lines represents then the error bounds of the coordinates of the measured point. It is clear that at small apex angles (<45º) the error is mostly in the direction perpendicular to the camera baseline, at medium apex angles (45-90º) the errors are about equal in all three directions, and at large apex angles (>90º) the error is increasingly in the direction parallel to the camera baseline.

The test showed the optimum accuracy was at the 75º apex angle where the standard deviation in all three coordinates was 0.0013", however the curve is essentially flat from 45 to 90º which is the recommended operating range. The standard camera setup described in the next section ensures that the apex angles for all points in the measurement are well within the 45 to 90º bands where the accuracy is the best and relatively uniform.

The differences between the first 5 measurements and the last 5 measurements at each apex angle were also analyzed to determine if the camera movement caused any degradation in accuracy. As expected, no detectable degradation was found.

Probe Test

For the probe accuracy test, fifty nest targets were used. The targets were distributed in all three dimensions throughout the test volume. The probe was fitted with a 1.5 inch diameter sphere tip and the tip location was calibrated using the standard field calibration procedures and equipment provided with the system. As mentioned earlier, the laser tracker also measured the nest points using a 1.5 inch diameter corner cube reflector. Using the same size sphere for both systems simplified the measurement and analysis. The cameras were located according to the standard placement rules described in the V-STARS manual, namely:

- Place the two cameras so that one camera is just outside the left boundary of the measured area, and the other is just outside its right edge.
- The cameras should be placed back far enough from the object so you can comfortably fit the entire measured area in either camera's viewfinder. In general, this means you will be back about the same distance as the size of the measured area.
- Aim the cameras so the measured area of the object is approximately centered in the viewfinder.

A diagram of typical camera placement is shown in Figure 11. Following the camera setup instructions above ensures that the apex angle to the probe targets will be within the recommended range of 45 to 90º.

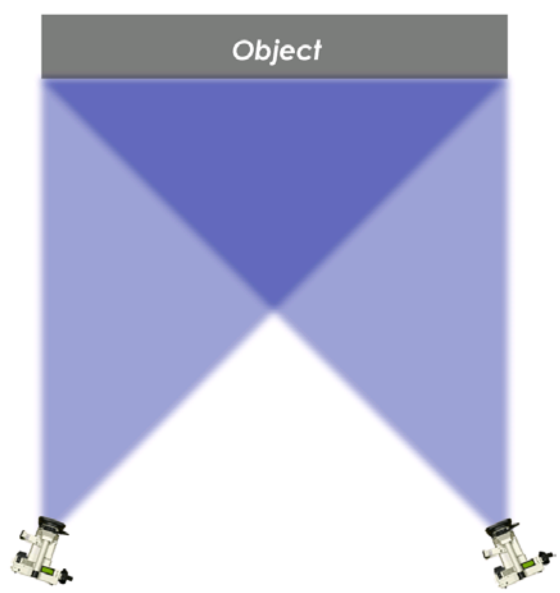

Figure 11: M-mode General Camera Setup

For the probe test, the camera apex angle was approximately 55º, well within the recommended range but significantly less than the optimum of 75º. The standard setup instructions sacrifice some potential accuracy in the direction perpendicular to the camera baseline (the Y direction in this case). However, this simplifies the setup (placing cameras at the left and right boundary of the object is far easier to specify than an apex angle of 75º), and makes probe measurement easier (the wider the apex angle the more difficult it is to aim the probe so that the targets on the probe are measurable by both cameras). In cases where the highest accuracy is desired, the camera apex angle can be set closer to 75º. Probe measurement will not be quite as easy but certainly tolerable.

The 50 nest points were measured sequentially, and their measurement was repeated ten times for a total of 500 measurements. Again, the unstable measurement mode was used and the same 25 well-distributed reference targets used for the apex-angle test were used to calculate the position and orientation of the cameras for each of the 500 probe measurements.

As in the apex-angle test, the cameras were moved slightly for the last five measurements of the nest points to check whether there was any degradation of accuracy due to camera movement.

Probe test results

For the analysis of the probe test results, the differences between the 50 laser-tracker values for the nest points and all their measured values were computed for each coordinate (X,Y,Z). Although, the probe measurements were already in the laser-tracker coordinate system due to the V-STARS/S survey of the 25 unstable mode reference points, the measurements were also transformed directly into the laser-tracker reference system using a 6 parameter rigid body transformation (scale was not allowed to adjust) so the evaluation would be consistent with what was done for other measurement systems that were tested.

The RSS of the three coordinate differences was also calculated for each measurement. The probe accuracy test results are presented in Table 1 and Figure 12.

Table 1. Statistics for Probe Measurements (inches)

Statistic	X	Y	Z	RSS
Average	0.0000	0.0000	0.0000	0.0018
Standard Deviation	0.0009	0.0016	0.0008	0.0010
Maximum	0.0025	0.0063	0.0037	0.0065
Minimum	-0.0025	-0.0041	-0.0021	0.0002
Range	0.0050	0.0105	0.0058	0.0063

The table list several statistics for the 500 measured points. Note the standard deviation for the Y coordinate differences is significantly larger than for X and Z. This is due to the small apex angle of the setup. As before, no detectable degradation of accuracy was found due to the camera movement on the last five measurements.

The figure shows a series of frequency histograms for the differences in X, Y and Z and for their RSS. Each of the X, Y and Z histograms is roughly normally distributed and centered at zero. The X and Z distributions are similar. The Y distribution is more spread out. This again further illustrates the larger overall differences for the Y measurements due to the fairly small apex angle. For the accuracy analysis that follows, we will assume the coordinate differences are normally distributed, and centered at zero.

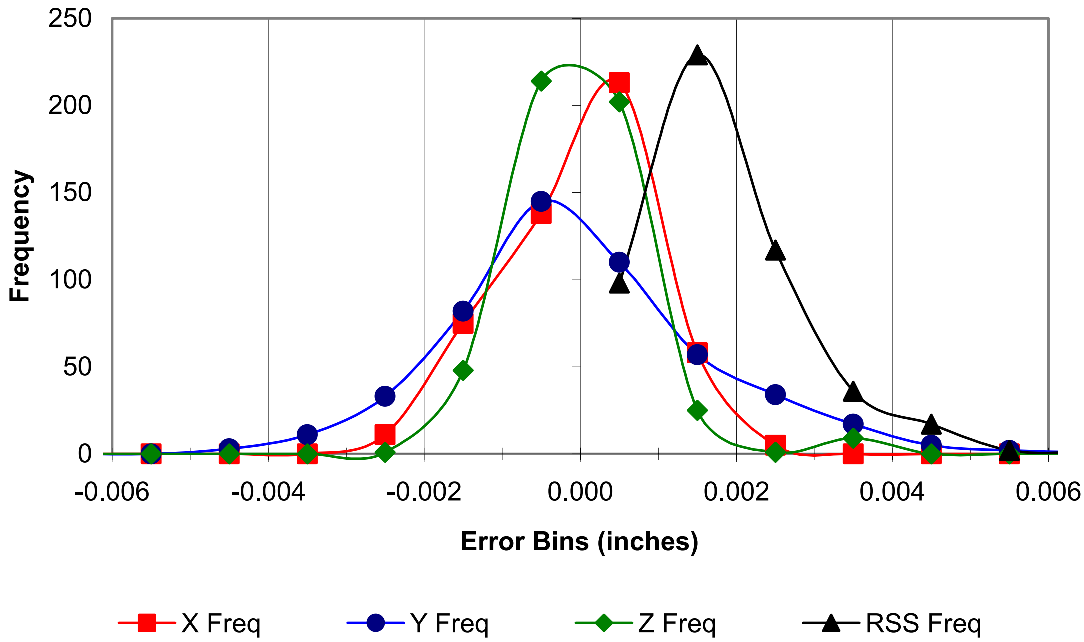

Figure 12. Histograms for Probe differences in X, Y, Z and for the RSS of the differences

The RSS distribution is not normally distributed. For example, it is always a positive number since it is the square root of a sum of squares. It is also not symmetric; it cannot be less than zero, but can be large so it is skewed to the right. Despite the fact it does not have the well known normal distribution, the RSS value is convenient because it is a single value that can be used to evaluate the accuracy of a system. As we shall see we can still use straightforward statistics such as the average and standard deviation to assess the accuracy of this distribution. We now proceed with a discussion of the accuracy achieved in the testing.

Test Accuracy Analysis

Before discussing the accuracy of the test results, some background information is helpful. Whenever one discusses the accuracy of a system, the accuracy cited is a statement of probability not certainty. Thus, when the accuracy of a measurement system is tested, the results are usually given as an accuracy level that was achieved with a certain level of probability. Since it is typical to assume that the measurement errors are normally distributed, the accuracies achieved are usually given at the 67% or 95% probability level, because these correspond very closely with the probability coverage of the normal distribution at one and two standard deviations from the average of the distribution, respectively. The term sigma is used for the standard deviation of a normal distribution. So, for example, when the accuracy is stated to be "0.001 inch at the two sigma level" that means the accuracy of a measurement will, on average, be within +/- 0.001 inch of the true value 95% of the time. Often the notation U67 and U95 is used to represent the 67% and 95% probability levels. The U stands for uncertainty.

NOTE: Since the RSS distribution is not a normal distribution the term "sigma" does not apply, so it will not be used for the RSS uncertainty quantity. Instead, the reported results for the RSS are in terms of the probability points (in our case we use the 95% probability point) of the error distributions.

Also, the conditions under which the test was conducted will affect the accuracy. This needs to be specified and understood so that one can reliably assess what the accuracy will be under other conditions. Furthermore, the ways in which accuracy can be specified can vary significantly. For example, GSI typically specifies the accuracy of its systems by separately analyzing the accuracy of the individual coordinates (X,Y, Z) while Boeing typically analyzes the accuracy of just the RSS component. To be complete, we will provide both analyses here.

In photogrammetry, accuracy is proportional to the size of the measured object. By specifying the accuracy as a proportion, the accuracy of the system can generally be applied to other size objects. Here, we shall convert the absolute accuracies achieved to parts per million (ppm). We use the maximum diagonal dimension of the object (called Lmax) as the size of the object used in the ppm calculations. For these tests, the measuring envelope was 141 inches in X, 39 inches in Y, and 55 inches in Z so Lmax is the RSS of these three values, which is 157 inches.

We will evaluate the accuracy at the 95% probability level. This allows us to use well-understood statistics such as the average and standard deviation to predict the 95% probability level reasonably well. The 95% probability level can be reasonably well approximated using a derived statistic that is two times the standard deviation of the differences plus the absolute value of the average of the differences. The use of this derived statistic is explained more fully in Appendix A.

Apex Angle Test Accuracy Analysis

The accuracy analysis for the apex angle tests is shown in the tables and figure below. Tables 2a through 2d list the average, standard deviation and U95 accuracy levels achieved for the X, Y and Z coordinates, and for the RSS of the three coordinates, respectively, for all seven apex-angle tests from 15 to 105º. Figure 13 plots the U95 accuracy in parts per million (ppm) for X, Y, Z and the RSS for the seven apex-angle tests.

Table 2a. X Accuracy Statistics

Degrees	15	30	45	60	75	90	105
Average (inches)	0.0003	0.0001	0.0001	-0.0001	0.0001	0.0001	0.0003
Standard Deviation (inches)	0.0021	0.0016	0.0014	0.0013	0.0013	0.0013	0.0019
U95 Accuracy (inches)[1]	0.0045	0.0033	0.0029	0.0025	0.0026	0.0027	0.0040
U95 Accuracy (ppm)[2]	28.7	21.3	18.4	15.9	16.8	17.4	25.3

Table 2b. Y Accuracy Statistics

Degrees	15	30	45	60	75	90	105
Average (inches)	-0.0005	-0.0004	-0.0007	-0.0002	-0.0002	-0.0004	-0.0005
Standard Deviation (inches)	0.0051	0.0034	0.0018	0.0016	0.0013	0.0013	0.0014
U95 Accuracy (inches)[1]	0.0107	0.0072	0.0044	0.0033	0.0029	0.0030	0.0032
U95 Accuracy (ppm)[2]	68.1	46.0	27.7	21.1	18.3	18.8	20.5

Table 2c. Z Accuracy Statistics

Degrees	15	30	45	60	75	90	105
Average (inches)	0.0006	0.0003	0.0003	0.0002	0.0001	0.0005	0.0002
Standard Deviation (inches)	0.0015	0.0015	0.0012	0.0013	0.0013	0.0014	0.0016
U95 Accuracy (inches)[1]	0.0037	0.0033	0.0028	0.0029	0.0028	0.0032	0.0034
U95 Accuracy (ppm)[2]	23.3	21.2	17.6	18.5	17.7	20.5	21.5

Table 2d. RSS Accuracy Statistics

Degrees	15	30	45	60	75	90	105
Average (inches)	0.0045	0.0035	0.0025	0.0022	0.0021	0.0021	0.0025
Standard Deviation (inches)	0.0036	0.0021	0.0011	0.0010	0.0009	0.0010	0.0014
U95 Accuracy (inches)[1]	0.0117	0.0077	0.0047	0.0043	0.0039	0.0042	0.0052
U95 Accuracy (ppm)[2]	74.7	48.9	29.9	27.4	25.1	26.9	33.4

[1] U95 absolute accuracy = Absolute value(Average) + 2 * Standard Deviation

[2] U95 proportional accuracy = 1,000,000 * U95 Accuracy (inches) / 157" (Lmax)

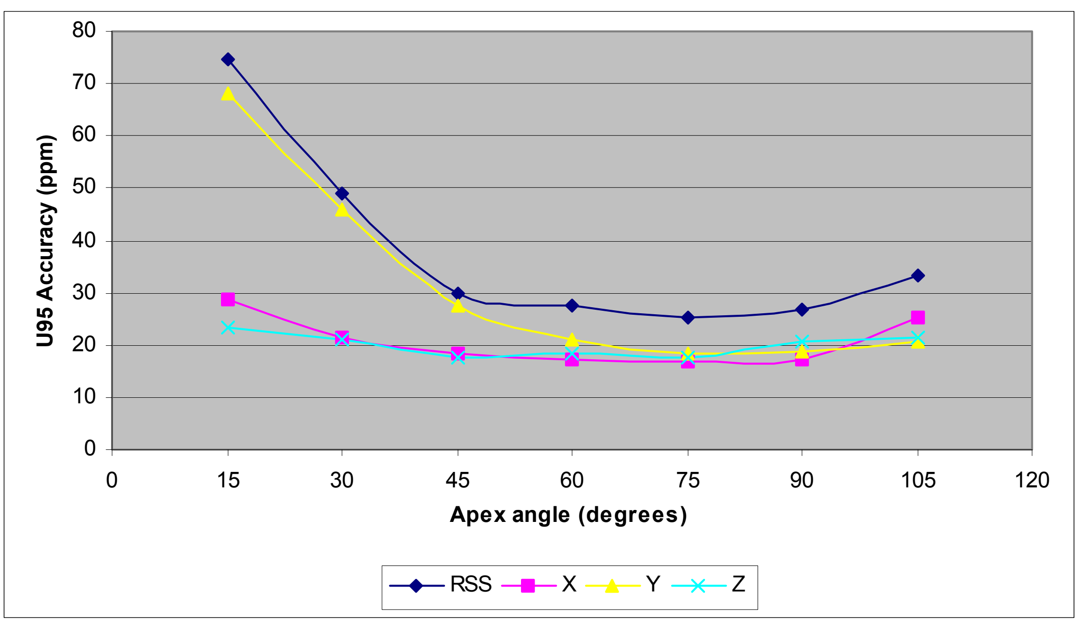

Figure 13. Apex Angle Accuracy in Parts per Million

The U95 accuracy for X and Z is 20 parts per million (ppm) or better for the entire recommended apex angle range of 45 to 90º, and is less than 30 ppm for the entire range of 15 to 105º. The Y accuracy is 21 ppm or better from 60 to 90º, and less than 30 ppm at 45º. The RSS is less than 30 ppm over the recommended range from 45 to 90º, and is the smallest at 75º where it is 25 ppm.

If the highest accuracy is desired in all three dimensions an apex angle near 75º should be used. When the accuracy in the direction perpendicular to the camera baseline is not so critical smaller apex angles can be used. However, apex angles below 45º should be avoided.

Probe Test Accuracy Analysis

The accuracy analysis for the apex angle tests is shown in table 3 below. The table lists the average, standard deviation and U95 accuracy levels achieved for the X, Y and Z coordinates, and for the RSS of the three coordinates.

Table 3. Probe Accuracy Statistics

Statistic	X	Y	Z	RSS
Average (inches)	0.0000	0.0000	0.0000	0.0018
Standard Deviation (inches)	0.0009	0.0016	0.0008	0.0010
U95 Accuracy (inches)[1]	0.0018	0.0032	0.0017	0.0038
U95 Accuracy (ppm)[2]	11.5	20.4	10.8	24.2

[1] U95 absolute accuracy = Absolute value(Average) + 2 * Standard Deviation

[2] U95 proportional accuracy = 1,000,000 * U95 Accuracy (inches) / 157" (Lmax)

The U95 accuracy is better than 12 ppm in X, and Z, and is 20 ppm in Y. The RSS accuracy is better than 25 ppm. The Y accuracy is worse than the X and Z accuracy primarily because the apex angle for the probe testing was approximately 55º.

Best Practices

As a result of this testing, some rules of best practice can be established to ensure accurate results are consistently achieved. These are:

1. When possible keep the apex angles between 45 and 90º. When the highest accuracy is desired in all three directions, the apex angle should be 60 to 90º. The standard camera setup instructions ensure the apex angle is well within 45 to 90º.
2. In the unstable mode of operation, there should be at least 16 well-distributed control points on the object. These points should be more accurate than the desired accuracy for the V-STARS/M measurement. Typically this is accomplished using a highly redundant V-STARS/S survey of the object.
3. Any targets used - whether they be individual targets or targets on the probes - should be clean and undamaged. The M system includes checks of the targets, and if these checks indicate a problem with a target, the target should be checked and replaced if necessary.
4. The probe calibration should be checked before a measurement proceeds. The M system can be used to quickly check a probe's calibration at the outset of a measurement. If a probe is found to be out of tolerance, it should be re-calibrated.
5. The ambient temperature variation of the object should be less than 4º Fahrenheit during the survey.

Summary

This examination of the GSI M-mode system was composed of two individual tests. Each test focused on specific attributes of the system. When considered together, these tests provide an effective analysis of the system's performance when used in factory applications. The tests were:

1. The Apex-Angle test, which determined the effect of camera apex angle on accuracy, and also determined the accuracy of measurement of individual target points.
2. The Probe test, which determined the system's accuracy measuring points with the hand-held probe.

The results of the test can be summarized as follows:

1. The Apex Angle test results showed that the V-STARS/M system's accuracy is relatively constant over the recommended operating range of 45 to 90º. Outside these ranges, accuracy degrades measurably, primarily in the direction perpendicular to the camera baseline for apex angles less than 45º, and primarily in the direction parallel to the camera baseline for apex angles greater than 90º. U95 RSS uncertainty is less than 30 parts-per-million (ppm) for apex angles from 45 to 90º, and is best at 75º where it is 25 ppm. U95 uncertainties for the individual directions (X, Y, Z) are generally better than 20 ppm over the recommended apex angle operating range.
2. The Probe Test showed that the V-STARS/M system produced U95 RSS uncertainty measurements of 24 ppm using the hand-held probe. The U95 uncertainty in the direction perpendicular to the camera baseline was 20 ppm, and was better than 12 ppm in the other two directions. The poorer accuracy in the one direction was due to the small apex angle for the probe test.
3. Both tests confirmed that in unstable mode, movement of the cameras has no detectable effect on accuracy.

The caveats are that the system must be used according to a set of best practices to achieve these results. These practices are described in the body of the report. They include:

1. the apex angle between cameras and probe should be between 45 and 90º, and should be between 60 and 90º when the best accuracy in all three directions is desired.
2. in the unstable mode of operation, there should be at least 16 well-distributed, well-known control points on the object,
3. the ambient temperature variation of the object should be less than 4ºF during the survey.

Acknowledgements

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References

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Appendix A. Accuracy Statistics and U95 Uncertainty

A common way to assess the accuracy of a measurement against a known (and presumed perfect) standard is to compute the standard deviation of all the measured differences from the standard values. Then, if the confidence level at a given probability is desired, one can multiply the computed standard deviation by the number of standard deviations of the normal distribution curve at the desired probability.

This technique relies on two key assumptions. First, that the measurements differences are normally distributed, and second, that they have zero bias (that is, the average of the differences is zero). Often, however, the data may be normally distributed but does not have zero bias. This case is illustrated in the figures below for a set of measurements that is normally distributed but with an average difference (bias) that is not zero.

Two cases of normal distributions both with a standard deviation of one are shown in Figures A1 and A2 below for illustration. In the left figure, the bias is zero, while in the right figure the bias is positive 1. The two sigma (U95) bounds are marked by the dotted lines.

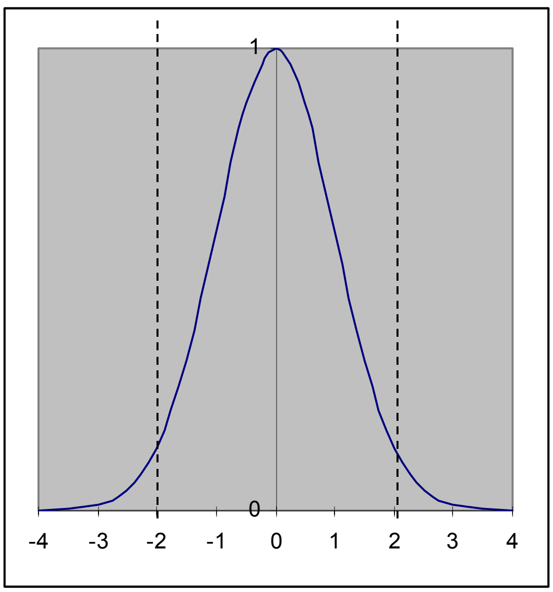

Figure A1. Unbiased normal distribution

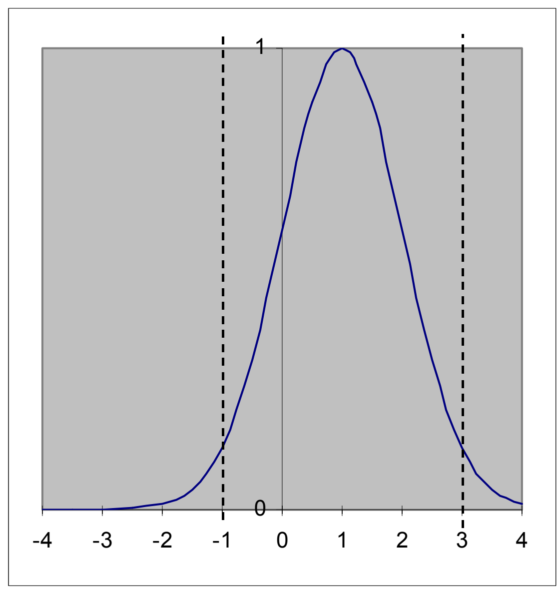

Figure A2. Biased normal distribution

In the examples shown in Figure A1, the standard deviation alone can be used to estimate the upper and lower accuracy bounds, but this is not the case for the example in figure A2. Instead, one must use the bias value (the average) together with the standard deviation to predict the upper and lower bounds for the desired probability level. So in the first example the U95 bounds are +/- 2, but in the second example, they are –1,+3. The spread of the bounds is the same since the standard deviations are identical, but the values of the bounds are not because the bias has shifted the distribution.

Although modifying the U95 bounds by the bias of the measurements is correct, it means one now has two different numbers specifying the accuracy range instead of one. To avoid this, one can simply use the bound number with the largest absolute value as a +/- bound for the U95 accuracy (for example, +/-3 in figure A2 above). This "single-ended" value will then be a conservative accuracy estimate since more of the probability curve will be encompassed.

For simplicity, we have chosen to provide single-ended U95 values for this report. Since we have assumed the X, Y, and Z differences are normally distributed, we add the absolute value of the bias to two times the standard deviation to get the U95 uncertainty value.

As mentioned in the report, the RSS distribution not only has a non-zero bias; it also is not a normal distribution (it can not be less than 0, and it is skewed to the right). It turns out, the U67 and U95 points of this distribution are very difficult to calculate analytically and require knowledge of the distributions of the component values (the X, Y and Z distributions) of the RSS distribution. However, it turns out the same derived statistic used for the normal distributions is a reasonable approximation for the U95 uncertainty value for the RSS distribution. The complete explanation for this is beyond the scope of this report, however, Figure A3 below helps show why this is so. The figure shows a typical RSS distribution.

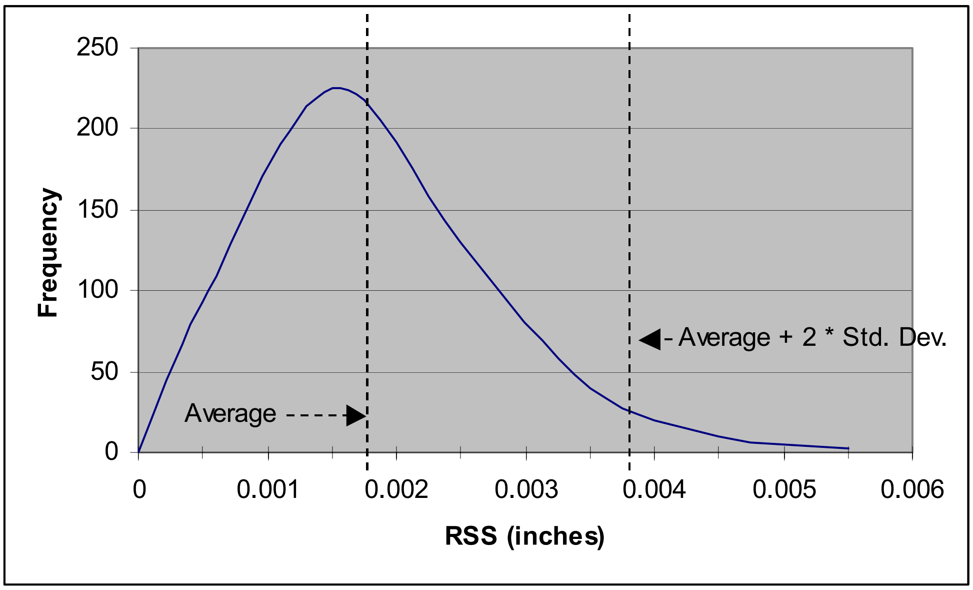

Figure A3. Sample RSS distribution

The figure shows the average of the RSS distribution, which is slightly past the peak due to the rightward skew of the distribution. The average value of the RSS distribution approximately represents the 50% point of the distribution; that is approximately 50% of the points are less than the average, and 50% of the points are higher than the average. (Although the median is the actual point 50% point by definition, it turns out to be more accurate, and more conservative, to use the average for our U95 calculation). Also notice that the RSS distribution looks similar in shape to the normal distribution. Although it is skewed to the right, it does peak near the middle and falls off at the edges. Therefore, the standard deviation of the RSS distribution provides a good approximation for the probability distribution of the RSS. (That is, approximately 67% of the RSS points lie within +/-1 standard deviation of the average; approximately 95% of the points lie within +/-2 standard deviations of the average, etc.)

If we assume the average is approximately the 50% point of the RSS distribution, and two standard deviations above the average is approximately another 47.5% of the distribution (half of 95%), then the average plus two times the standard deviation (which is also shown in the figure) is approximately the 97.5% point of the distribution. Since this is close to 95% and somewhat higher, the value is somewhat conservative. Therefore, we use the same derived statistic for the U95 level for the RSS distribution that we use for the component values (X, Y and Z).